Overview of Mahadev's Protocol for Classical Verification of Quantum Computation

PhD Interview - Technical Talk

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Quantum Computations

Quantum computation give an advantage over classical computation in

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- Optimization
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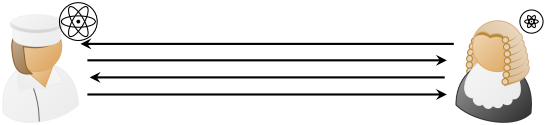
Verifying quantum computations classically is not feasible
$\Rightarrow$ Mahadev's Protocol: Use cryptography and interact classically with the quantum computer

Short History Lesson

- 2004: Question whether a classical computer can verify the result of a quantum computation through interaction is raised.
- BQP $\subseteq$ PSPACE $=$ IP, but powerful prover
- What if the Prover has to be efficient?
- Approach 1: Verifier has access to small quantum computer (error-correcting codes)



- Approach 2: Play multiple provers against each other (CHSH)
- Can we verify by only interacting with one prover without small quantum computer?

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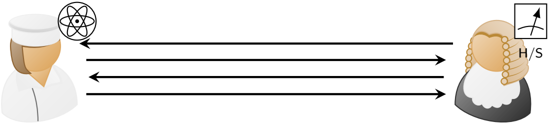
  1. Verifier reduces x to local Hamiltonian H_x
  2. Verifier requests state from prover
  3. Verifier checks if received state has low energy with respect to H_x. If energy is low, Verifier accepts.



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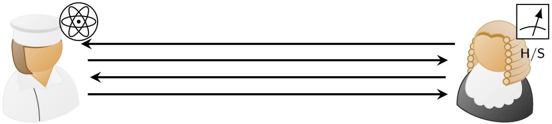
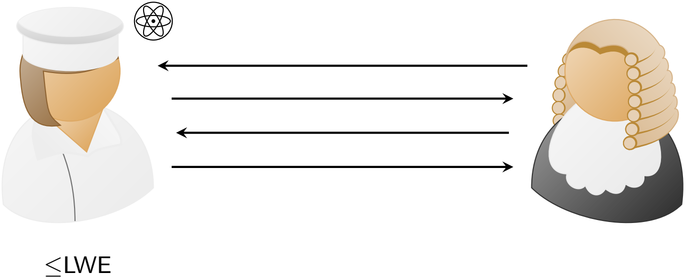


- What if we don't have access to trusted measurement device?

Mahadev's Protocol - Overview



- Measurement protocol: Classical verifier (BPP) using q. prover (BQP) as trusted measurement device
- Forces Prover to:
  - construct n qubit state of her choice
  - measure each qubit in Hadamard or Standard basis
  - report measurement result to verifier
- Soundness enforced based on LWE assumption: If verifier accepts, there exists a quantum state underlying the measurement result that is independent of the verifier's measurement choice

Commitment Phase

Definition (TCF+)

A function family $\mathcal{F} = \{f_{i,0}, f_{i,1} : \mathcal{X} \to \mathcal{D}\}$ is called TCF+ if

- there exists ppt $\mathsf{Gen}_{\mathcal{F}}$: $(i, \mathsf{td}_i) \leftarrow \mathsf{Gen}_{\mathcal{F}}(1^\lambda)$
- $f_{i,0}, f_{i,1}$ injective with same image
- there exists ppt Inv that given $i, \mathsf{td}_i, y \in \mathcal{D}$, finds both preimages: $(x_0, x_1) \leftarrow \mathsf{Inv}(i, \mathsf{td}_i, y)$
- adaptive Hardcore bit: $\forall d \neq 0 \forall$ claws (x_0, x_1) is is hard to compute both $d \cdot (x_0 \oplus x_1)$ and a preimage x_0 or x_1; $\exists\ d$ s.t. $\forall$ claws (x_0, x_1) the bit $d \cdot (x_0 \oplus x_1)$ is the same and indistinguishable from uniform

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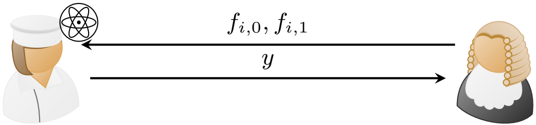

$\leq$LWE

- The Verifier samples TCF+ functions and sends $f_{i,0}, f_{i,1}$ to the Prover.
- Prover entangles a quantum state of his choice with a claw $y = f_{i,0}(x_0) = f_{i,1}(x_1)$ and sends y to the verifier

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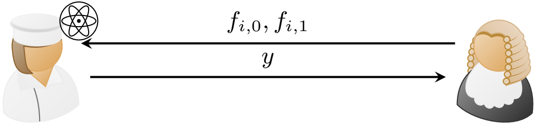

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$$|\psi\rangle = \sum_{b \in \{0,1\}} \alpha_b |b\rangle \to \sum_{x \in \mathcal{X}} \sum_{b \in \{0,1\}} \alpha_b |b\rangle |x\rangle |f_{i,b}(x)\rangle \xrightarrow{f_{i,b}(x) = y} \sum_{b \in \{0,1\}} \alpha_b |b\rangle |x_b\rangle = \text{Enc}(|\psi\rangle)$$

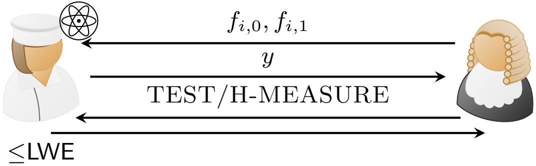
Challenge

TEST

- Verifier requests preimage (b, x_b) of y

$$\mathsf{Enc}(|\psi\rangle) = \sum_{b\in\{0,1\}} \alpha_b |b\rangle |x_b\rangle$$

$\Rightarrow$ Prover can measure in standard basis and sends result to the prover



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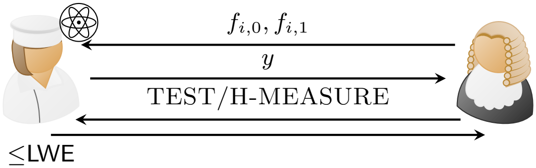
H-MEASURE

- Prover applies H to entire encoded state, measures second register and sends result r to the verifier

$$\mathsf{Enc}(|\psi\rangle) \xrightarrow{H} X^{d\cdot(x_0 \oplus x_1)} H |\psi\rangle$$

- Verifier decodes measurement by XORing $d \cdot (x_0 \oplus x_1)$ to r



Standard Basis Measurement

Definition (TIF+)

A function family $\mathcal{G} = \{g_{i,0}, g_{i,1} : \mathcal{X} \to \mathcal{D}\}$ is called TIF+ if

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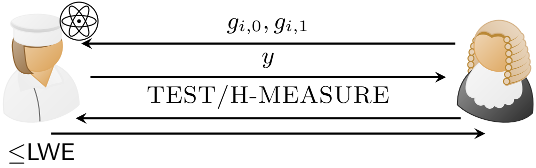
- $g_{i,0}, g_{i,1}$ injective with distinct images
- there exists ppt $\mathsf{Inv}_{\mathcal{G}}$ that given, $i, \mathsf{td}_i, y \in \mathcal{D}$ finds preimage $x \leftarrow \mathsf{Inv}_{\mathcal{G}}(i, \mathsf{td}_i, y)$
- $(f_{i,0}, f_{i,1})$ computationally indistinguishable from $(g_{i,0}, g_{i,1})$

This acts as standard basis measurement:

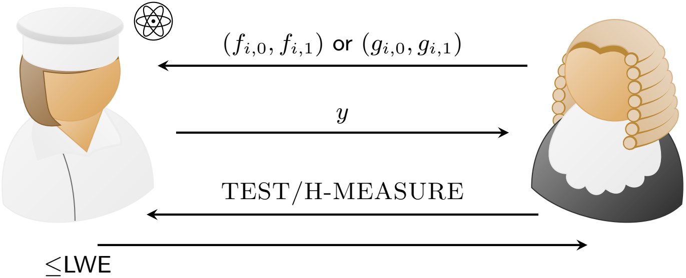
$$|\psi\rangle = \sum_{b\in\{0,1\}} \alpha_b |b\rangle \to \sum_{x\in\mathcal{X}} \sum_{b\in\{0,1\}} \alpha_b |b\rangle |x\rangle |g_{i,b}(x)\rangle$$

Given $y = g_{i,b}(x)$ the Verifier can reconstruct measurement result b using trapdoor



Verifier uses y to recover measurement result; ignores Hadamard measurement result

Protocol - Overview



Verifier chooses basis:

- Hadamard: send TCF+ $(f_{i,0}, f_{i,1})$
- Standard: send TIF+ $(g_{i,0}, g_{i,1})$

Verifier either:

- tests state structure or
- request measurement result

$\Rightarrow$ Apply this protocol for every qubit in parallel

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Completeness

- Reduce problem to k-LOCAL-HAMILTONIAN
- Verifier chooses measurement basis
- Prover commits to ground state
- Prover measures honestly and sends measurement result
- Verifier can deduce that the commited state has low enough energy

Soundness

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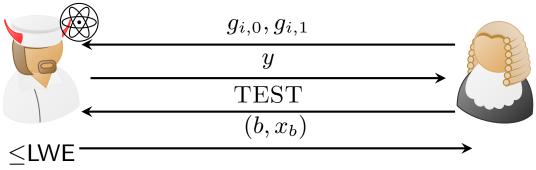

Definition (Pauli Twirl (Informal))

- Conjugation of unitary U by random Pauli
- $(X^x Z^z)^\dagger U (X^x Z^z)$
- averaging over random Paulis $\Rightarrow$ effect of Pauli

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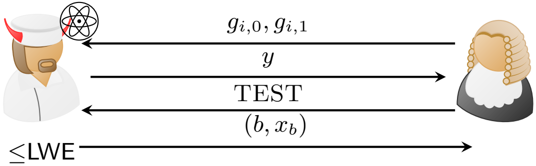

- Prover's state must have been of the form:

$$\sum_{b \in \{0,1\}} \alpha_b |b\rangle |x_b\rangle |\psi_{b,x_b}\rangle \text{ or } |b\rangle |x_b\rangle |\psi_{b,x_b}\rangle$$

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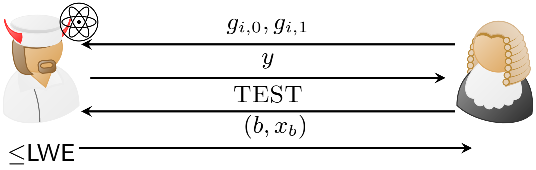

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- Verifier's decoding is $d \cdot (x_0 \oplus x_1)$
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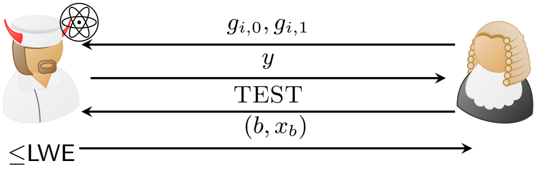

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- Let U be the deviation from the protocol
- Verifier's decoding is $d \cdot (x_0 \oplus x_1)$
- Part of U acting on first register computationally randomized by initial state and Verifier's decoding
- $d \cdot (x_0 \oplus x_1)$ computationally indistinguishable from uniform $\Rightarrow$ Use Pauli Twirl and U is simplified to Pauli
- U commutes with standard basis measurement $\Rightarrow$ U could have been applied before the commitment $\Rightarrow$ measurement distribution equivalent to honest prover with commited state $|\psi'\rangle = U|\psi\rangle$

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- ↯ Polynomially many repetitions needed

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[Ala+20] Non-interactive classical verification of quantum computation

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- Use a parallel repetition theorem to run $3\text{poly}(\lambda)$ steps in 3 steps
- Fiat-Shamir $\Rightarrow$ Non-interactive (QROM)
- classical NIZK + classical FHE $\Rightarrow$ Zero-Knowledge (requires circuit-private FHE)

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[Bar+22] Succint Classical Verifcation of Quantum Computation

- Succint Key Generation based on iO / PPRF
- SNARGs in QROM

[Ala+20] Non-interactive classical verification of quantum computation

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2. Parallel repetition

   $\lightning$ Private coin, rewinding (nested rejection sampling)
   - For NO instance: path of Verifier for two challenges correspond to nearly computational orthogonal projectors
   - k-fold parallel repetition: each pair of distinct challenge tuples correspond to nearly orthogonal projectors
   - Prover can only succeed in negligible fraction of challenge strings
   - $\delta \rightarrow \delta^k$

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4. Fiat-Shamir
   - $c = \mathcal{H}(H_x, \mathsf{pk}, y)$
   - QROM

Succint classical verification of quantum computation

1. "Succint batch key generation algorithm"
    - outputs short description of many $(\mathsf{pk}, \mathsf{sk})$ pairs
    - can be constructed from iO + PPRFs
    - compose succint key generation with TCF+
2. provides template for succint arguments for QMA

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  - Prover measures and sends result to Verifier
- Complete, Soundness (reduce perfect attackers to trivial attackers)
- Application of parallel repetition, FS possible
- ZK possible
- Succint arguments with succint key generation based on iO / PPRF possible

References I

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